





Dr. Khalil Qatu

ENCE 331: Compressibility of soil

Loading and soil settlement

- A stress increase caused by the construction of foundations or other loads compresses soil layers.
- The compression is caused by
 - deformation of soil particles
 - Relocations of soil particles
 - Expulsion of water or air from the void spaces.



- soil settlement caused by loads may be divided into three broad categories:
 - Elastic settlement (or immediate settlement),

which is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Elastic settlement calculations generally are based on equations derived from the theory of elasticity.

• Primary consolidation settlement,

which is the result of a volume change in saturated cohesive soils because of expulsion of the water that occupies the void spaces.

• Secondary consolidation settlement,

which is observed in saturated cohesive soils and organic soil and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

$$S_T = S_c + S_s + S_e$$

- Elastic, or immediate, settlement of foundations (Se) occurs directly after the application of a load without a change in the moisture content of the soil
- The magnitude settlement depend on the flexibility of the foundation and the type of material on which it is resting on.
- In the previous chapter we learned how to determine the increase in stress due to additional loading on soil (which causes elastic settlement)
- These relationships are based on the following assumptions
 - The load is applied at the ground surface.
 - The loaded area is flexible.
 - The soil medium is homogeneous, elastic, isotropic, and extends to • a great depth.





Flexible





• Based on theory of elasticity, if the foundation is perfectly flexible, the settlement may be expressed as

$$S_e = \Delta \sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

where $\Delta \sigma$ = net applied pressure on the foundation

- μ_s = Poisson's ratio of soil
- E_s = average modulus of elasticity of the soil under the foundation measured from z = 0 to about z = 5B
- B' = B/2 for center of foundation
 - = B for corner of foundation
- I_s = shape factor (Steinbrenner, 1934)

$$I_f$$
 = depth factor (Fox, 1948) = $f\left(\frac{D_f}{B}, \mu_s, \text{and } \frac{L}{B}\right)$

 α = factor that depends on the location on the foundation where settlement is being calculated

ettlement at the	settlement at the
center of the	<i>corner</i> of the
foundation	foundation
$\alpha = 4$	$\alpha = 1$

Foundation

$$B \times L$$

 $S_{e(rigid)} \approx 0.93S_{e(flexible, center)}$
Rigid
foundation
settlement
 $\mu_s = Poisson's ratio$
 $E_s = modulus of elasticity$
Soil
Rock

Weighted average

• Shape Factor (I_s)

The value of I_s depends on the soil properties, foundation dimensions, and the location where the settlement is being calculated settlement at the

$$I_{s} = F_{1} + \frac{1 - 2\mu_{s}}{1 - \mu_{s}}F_{2}$$

$$F_{1} = \frac{1}{\pi}(A_{0} + A_{1})$$

$$F_{2} = \frac{n'}{2\pi}\tan^{-1}A_{2}$$

$$A_{0} = m'\ln\frac{(1 + \sqrt{m'^{2} + 1})\sqrt{m'^{2} + n'^{2}}}{m'(1 + \sqrt{m'^{2} + n'^{2} + 1})}$$

$$A_{1} = \ln\frac{(m' + \sqrt{m'^{2} + 1})\sqrt{1 + n'^{2}}}{m' + \sqrt{m'^{2} + n'^{2} + 1}}$$

$$A_{2} = \frac{m'}{n'\sqrt{m'^{2} + n'^{2} + 1}}$$

ettlement at the center of the foundation $m' = \frac{L}{B}$ $n' = \frac{H}{\left(\frac{B}{2}\right)}$





• Depth Factor (I_f)

The value of I_s depends on the soil properties, foundation dimensions, and depth of foundation

Table 11.3 Variation of I_f with L/B and D_f/B

		I_f				
L/B	D_f/B	$\mu_{\rm s}=0.3$	$\mu_{\rm s}=0.4$	$\mu_{\rm s}=0.5$		
1	0.5	0.77	0.82	0.85		
	0.75	0.69	0.74	0.77		
	1	0.65	0.69	0.72		
2	0.5	0.82	0.86	0.89		
	0.75	0.75	0.79	0.83		
	1	0.71	0.75	0.79		
5	0.5	0.87	0.91	0.93		
	0.75	0.81	0.86	0.89		
	1	0.78	0.82	0.85		



• Typical soil properties:

Table 11.4Representative Values of the
Modulus of Elasticity of Soil

	E_s			
Soil type	kN/m ²	lb/in. ²		
Soft clay	1800-3500	250-500		
Hard clay	6000–14,000	850-2000		
Loose sand	10,000–28,000	1500-4000		
Dense sand	35,000–70,000	5000-10,000		

Table 11.5Representative Values of
Poisson's Ratio

Type of soil	Poisson's ratio, μ_{s}			
Loose sand	0.2-0.4			
Medium sand	0.25-0.4			
Dense sand	0.3-0.45			
Silty sand	0.2-0.4			
Soft clay	0.15-0.25			
Medium clay	0.2–0.5			

• Example:

A rigid shallow foundation 1m x 1m in plan is shown in the figure. Calculate

the elastic settlement at the center of the foundation.

$$S_e = \Delta \sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$



- When a saturated soil layer is subjected to a stress increase, the pore water pressure is increased suddenly.
- In sandy soils that are highly permeable, the drainage caused by the increase in the pore water pressure is completed immediately.
- Pore water drainage is accompanied by a reduction in the volume of the soil mass, which results in settlement
- Rapid drainage of the pore water in sandy soils, elastic settlement and consolidation occur simultaneously
- Saturated compressible clay layer is subjected to a stress increase, because hydraulic conductivity of clay is significantly smaller than that of sand, excess pore water pressure generated by loading gradually dissipates over a long period.
- the consolidation settlement in the clay may continue long after the elastic settlement





9

- One-dimensional consolidation test
 - The procedure was suggested by Terzaghi to quantify this type of settlement
 - The specimen is loaded as shown and the deformation (settlement) is observed over time
 - Each load usually is kept for 24 hours. After that, the load usually is doubled, which doubles the pressure on the specimen.
 - At the end of the test, the dry weight of the test specimen is determined
 - The deformation of the specimen against time for a given load increment
 - Settlement is observed in three stages:
 - Initial compression
 - Primary consolidation
 - during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water
 - Secondary consolidation

which occurs after complete dissipation of the excess pore water pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric



Porous stone Soil specimen Specimen ring





- One-dimensional consolidation test
 - Calculate the height of solids, H_s
 - Calculate the initial height of voids $H_v = H H_s$
 - Calculate the initial void ratio $e_0 = \frac{V_v}{V} = \frac{H_v A}{H} = \frac{H_v}{H}$
 - For the first incremental loading, σ_1 (total load/unit area of • specimen), which causes a deformation $\Delta H_1 \rightarrow$ calculate the $\Delta e_1 = \frac{\Delta H_1}{H}$ change in the void ratio
 - Calculate the new void ratio after consolidation caused by the pressure increment as $e_1 = e_o - \Delta e_1$
 - For the next loading which causes additional deformation ΔH_2 , the • void ratio at the end of consolidation is calculated.
 - The effective stress σ ' and the corresponding void ratios (e) at the end of consolidation are plotted on semilogarithmic graph paper.



Effective pressure, σ' (log scale)

 $H_{s} = \frac{W_{s}}{AG_{s}\gamma_{sr}} = \frac{M_{s}}{AG_{s}\rho_{sr}}$

- One-dimensional consolidation test
 - Effect of Pressure History on consolidation settlement
 - A soil in the field at some depth has been subjected to a certain maximum effective past pressure in its geologic history
 - This maximum effective past pressure may be equal to or less than the existing effective overburden pressure at the time of sampling
 - During the soil sampling, the existing effective overburden pressure is also released, which results in some expansion
 - When this specimen is subjected to a consolidation test, a small amount of compression (that is, a small change in voic ratio) will occur when the effective pressure applied is less than the maximum effective overburden pressure in the field to which the soil has been subjected in the past
 - When the effective pressure on the specimen becomes greater than the maximum effective past pressure, the change in the void ratio is much larger, and the e-log- σ relationship is practically linear with a steeper slope.
 - This relationship can be verified in the laboratory by loading the specimen to exceed the maximum effective overburden pressure, and then unloading and reloading again.
 - This leads us to the two basic definitions of clay based on stress history
 - Normally consolidated
 - whose present effective overburden pressure is the maximum pressure that the soil was subjected to in the past
 - Over-consolidated

whose present effective overburden pressure is less than that which the soil experienced in the past. The maximum effective past pressure is called the pre-consolidation pressure.



Effective pressure, σ' (log scale)

- One-dimensional consolidation test
 - Effect of Pressure History on consolidation settlement
 - Pre-consolidation Pressure (σ_c')
 - Casagrande (1936) suggested a simple graphic construction to determine the preconsolidation pressure σ'_c from the laboratory e-log - σ plot.
 - The procedure is as follows
 - By visual observation, establish point a, at which the e-log σ plot has a minimum radius of curvature.
 - Draw a horizontal line ab.
 - Draw the line ac tangent at a.
 - Draw the line ad, which is the bisector of the angle bac.
 - Project the straight-line portion gh of the e-log σ plot back to intersect line ad at f.
 - The abscissa of point f is the pre-consolidation pressure, σ'_c .
 - Over consolidation ratio (OCR)

$$OCR = \frac{\sigma'_c}{\sigma'}$$



Pressure, σ' (log scale)

- One-dimensional consolidation test
 - Compression index and Swelling index
 - Compression index

It is the slope of the e-log - σ plot when $\sigma'_o > \sigma'_c$ and it is used in the calculation of field settlement caused by consolidation,

It is calculated graphically for Normally consolidated or Over-consolidated clays from e-log - σ plot as shown in the figures

 $C_c = 0.009(LL - 10) \qquad C_c = 0.141G_s^{1.2} \left(\frac{1 + e_o}{G_s}\right)^{2.38} \qquad C_c = 0.2343 \left[\frac{LL(\%)}{100}\right] G_s$

• Swelling index

It is the slope of the e-log - σ plot when $\sigma'_o < \sigma'_c$ It is used in the calculation of field settlement caused by consolidation for over consolidated clays

It is calculated graphically from unloading e-log - σ plot as shown in the figure $C_s \approx \frac{1}{5} \operatorname{to} \frac{1}{10} C_c$ $C_s = 0.0463 \left[\frac{LL(\%)}{100} \right] G_s$ $C_s \approx \frac{PI}{370}$



Figure 11.21 Consolidation characteristics of overconsolidated clay of low to medium sensitivity



Figure 11.20 Consolidation characteristics of normally consolidated clay of low to medium sensitivity

height of

specimer

Specimen area = A Void Solid

- Primary consolidation settlement
 - Since the slope of the consolidation curve is different for Normally consolidated from over consolidated clays
 - For Normally consolidated clays



For Over consolidated clays •

If
$$\sigma'_o + \Delta \sigma' \le \sigma'_c$$

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \sigma'_o}{\sigma'_o}\right)$$

• If $\sigma'_o + \Delta \sigma' > \sigma'_c$

•

$$S_c = \frac{C_s H}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log \left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_c} \right)$$



Specimen area = A Void Solid



Dr. Khalil M. Qatu

represent ??

• Primary consolidation settlement

Example: If a uniformly distributed load, $\Delta \sigma$ is applied at the ground surface, what is the settlement of the clay layer caused by primary consolidation if:

- The clay is normally consolidated
- The pre-consolidation pressure, $\sigma'_c = 200 \text{ kN/m}^2$
- $\sigma_c' = 150 \text{ kN/m}^2$



 $\gamma_{\text{sat(clay)}} = \frac{(G_s + e)\gamma_w}{1 + e}$ $C_c = 0.141G_s^{1.2} \left(\frac{1 + e_o}{G}\right)^{2.38}$

 $C_s \simeq \frac{1}{5} \operatorname{to} \frac{1}{10} C_c$

• Primary consolidation settlement

Example: If a uniformly distributed load, $\Delta \sigma$ is applied at the ground surface, what is the settlement of the clay layer caused by primary consolidation if:

- The clay is normally consolidated
- The pre-consolidation pressure, $\sigma'_c = 200 \text{ kN/m2}$
- $\sigma_c' = 150 \text{ kN/m2}$



Sand Clay

- Secondary consolidation settlement (Creep)
 - This settlement is observed because of the plastic adjustment of soil fabrics.
 - Secondary consolidation settlement is more important than primary consolidation in organic and highly compressible inorganic soils.
 - In over-consolidated inorganic clays, The secondary compression index is very small and of less practical significance.

$$S_{s} = C'_{\alpha} H \log \left(\frac{t_{2}}{t_{1}}\right) \qquad C_{\alpha} = \frac{\Delta e}{\log t_{2} - \log t_{1}} = \frac{\Delta e}{\log (t_{2}/t_{1})}$$
$$C'_{\alpha} = \frac{C_{\alpha}}{1 + e_{p}}$$



Dr. Khalil M. Qatu



 e_p = void ratio at the end of primary consolidation H = thickness of clay layer

u

$$S_{c} = \frac{C_{c}H}{1 + e_{o}} \log\left(\frac{\sigma_{o}' + \Delta\sigma'}{\sigma_{o}'}\right) \qquad S_{s} = C_{\alpha}' H \log\left(\frac{t_{2}}{t_{1}}\right)$$

• Secondary consolidation settlement

Example: a normally consolidated clay layer in the field, the following values are given

Thickness of clay layer = 2.6 m Void ratio, $e_o = 0.8$ Compression index, $C_c = 0.28$ Average effective pressure on the clay layer, $\sigma'_o = 127 \text{ kN/m}^2$ $\Delta \sigma' = 47 \text{ kN/m}^2$ Secondary compression index, $C_{\alpha} = 0.02$



What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (Note: Time for completion of primary settlement = 1.5 years.)

$$S_{c} = \frac{C_{c}H}{1 + e_{o}} \log\left(\frac{\sigma_{o}' + \Delta\sigma'}{\sigma_{o}'}\right) \qquad S_{s} = C_{\alpha}' H \log\left(\frac{t_{2}}{t_{1}}\right)$$

• Secondary consolidation settlement

Example: a normally consolidated clay layer in the field, the following values are given

Thickness of clay layer = 2.6 m Void ratio, $e_o = 0.8$ Compression index, $C_c = 0.28$ Average effective pressure on the clay layer, $\sigma'_o = 127 \text{ kN/m}^2$ $\Delta \sigma' = 47 \text{ kN/m}^2$ Secondary compression index, $C_{\alpha} = 0.02$



What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (Note: Time for completion of primary settlement = 1.5 years.)

- Settlement under foundation
 - So far, the applied load was assumed to be uniformly distributed and infinite in all directions
 - We studied how to calculate the stress increase due to limited load (foundation) in the previous chapter
 - The average increase in the pressure below the center of the foundation
 - Assuming that the pressure increase varies parabolically, using Simpson's rule, $\Delta \sigma$

$$\Delta \sigma'_{\rm av} = \frac{\Delta \sigma'_t + 4\Delta \sigma'_m + \Delta \sigma'_b}{6}$$



- Time Rate of Primary Consolidation
 - So far, we didn't discuss How long would it take to complete the primary consolidation settlement.
 - This is especially important when dealing with clay soils



 $\Delta u = 0$

- Time Rate of Primary Consolidation
 - The degree of consolidation (the amount of consolidation settlement) is linked to the excess pore water pressure Δu .
 - The time taken to dissipate this pressure is linked to water flow out of the soil
 - When the excess pore water pressure $\Delta u = 0 \rightarrow \text{Consolidation}$ is complete
 - Terzaghi 1D consolidation theory correlates the excess water pressure with depth and time



Valve open

 $\Delta u < \frac{P}{A}$

Valve open

- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$c_v = \text{coefficient of consolidation} = k/(\gamma_w m_v)$$

 $m_v = \text{coefficient of volume compressibility} = a_v / (1 + e_o)$

 $a_v = \text{coefficient of compressibility } (a_v \text{ can be considered constant for a narrow range of pressure increase}) = \frac{\Delta e}{\Delta \sigma'}$



Void ratio

- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

• The solution for this partial differential equation is

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin\left(\frac{Mz}{H_{\rm dr}}\right) \right] e^{-M^2 T_v}$$

where
$$m = \text{an integer}$$

 $M = (\pi/2)(2m + 1)$
 $u_o = \text{initial excess pore water pressure}$
 $T_v = \frac{c_v \iota}{H_{Ar}^2} = \text{time factor}$
 $c_v = \text{coefficient of consolidation} = k/(\gamma_w m_v)$



- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - The excess pore water pressure at any time is linked to the degree of consolidation U_z

$$U_{z} = \frac{u_{o} - u_{z}}{u_{o}} = 1 - \frac{u_{z}}{u_{o}}$$

where u_{z} = excess pore water pressure at time t .
$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^{2}} e^{-M^{2}T_{v}}$$
$$T_{v} = \frac{c_{v}t}{H_{dr}^{2}} = \text{time factor}$$
$$c_{v} = \text{coefficient of consolidation} = k/(\gamma_{w}m_{v})$$
$$H_{dr} = \text{Maximum drainage distance}$$
For $U = 0$ to 60%, $T_{v} = \frac{\pi}{4} \left(\frac{U\%}{100}\right)^{2}$
For $U > 60\%$, $T_{v} = 1.781 - 0.933$ log (100 - U%)
$$0 - \frac{1}{00} + \frac{1}{0} + \frac{1}$$

- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - Now, we can estimate the degree of consolidation for a given time factor (T_v) . (i.e., at a specific time)

lable 11.7	Variation	of T_v with U					
U (%)	T _v	U (%)	T _v	U (%)	T _v	U (%)	T _v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.54
2	0.0003	28	0.0615	54	0.230	80	0.56
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.84
13	0.0133	39	0.119	65	0.340	91	0.89
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.330
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		



- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - One-way vs two-way drainage



- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - Coefficient of consolidation (C_v)

D d_0 х **↑**C Deformation (increasing) х d_{100} d_0 F d_{50} d_{100} Α t_1 t_2 t_{50} Time (log scale)

Logarithm-of-time method



- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - Types of problems



• Find the time it takes to get a specific degree of consolidation (or settlement, portion of the total primary consolidation settlement)

From the Table, graph, or equations $U\% \to T_v = \frac{c_v t}{H_{dr}^2} \to c_v$, H_{dr} are given in the problem \to find t

- Find the degree of consolidation (or settlement, portion of the total primary consolidation settlement) after a given time Calculate $T_v = \frac{c_v t}{H_{dr}^2} \rightarrow c_v$, H_{dr} are given in the problem \rightarrow find t From the Table, graph, or equations U%
- In both types of problems, c_v can be calculated either

from the deformation vs. time curve

or from $c_v = \frac{k}{m_v \gamma_w}$, $m_v = \frac{a_v}{1+e}$, $a_v = \frac{\Delta e}{\Delta \sigma} = slope$

• Find the permeability of a soil given the degree of consolidation and time

From the Table, graph, or equations $U\% \rightarrow T_v = \frac{c_v t}{H_{dr}^2} \rightarrow calculate \ c_v \ , a_v, m_v \rightarrow k$

$$S_c = \frac{C_c H}{1 + e_o} \log \left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_o} \right) \qquad T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

- Time Rate of Primary Consolidation
 - Terzaghi 1D consolidation theory
 - Example:

A 4-m clay layer underlain by bedrock (hydraulic conductivity = 5×10^{-7} cm/s) in the field has a current effective stress of $\sigma' = 100$ kN/m². There is a net stress increase of $\Delta \sigma = 195$ kN/m² due to a foundation load. Only four data points are available from a consolidation test on the clay, as shown.

- Estimate the primary consolidation settlement of the clay layer in the field.
- How long would it take to reach settlement of 25cm.
- How long would it take to reach 99% degree of consolidation
- What is the total settlement after 1 year



- Accelerating Consolidation Settlement
 - Vertical drains & Pre-compression

